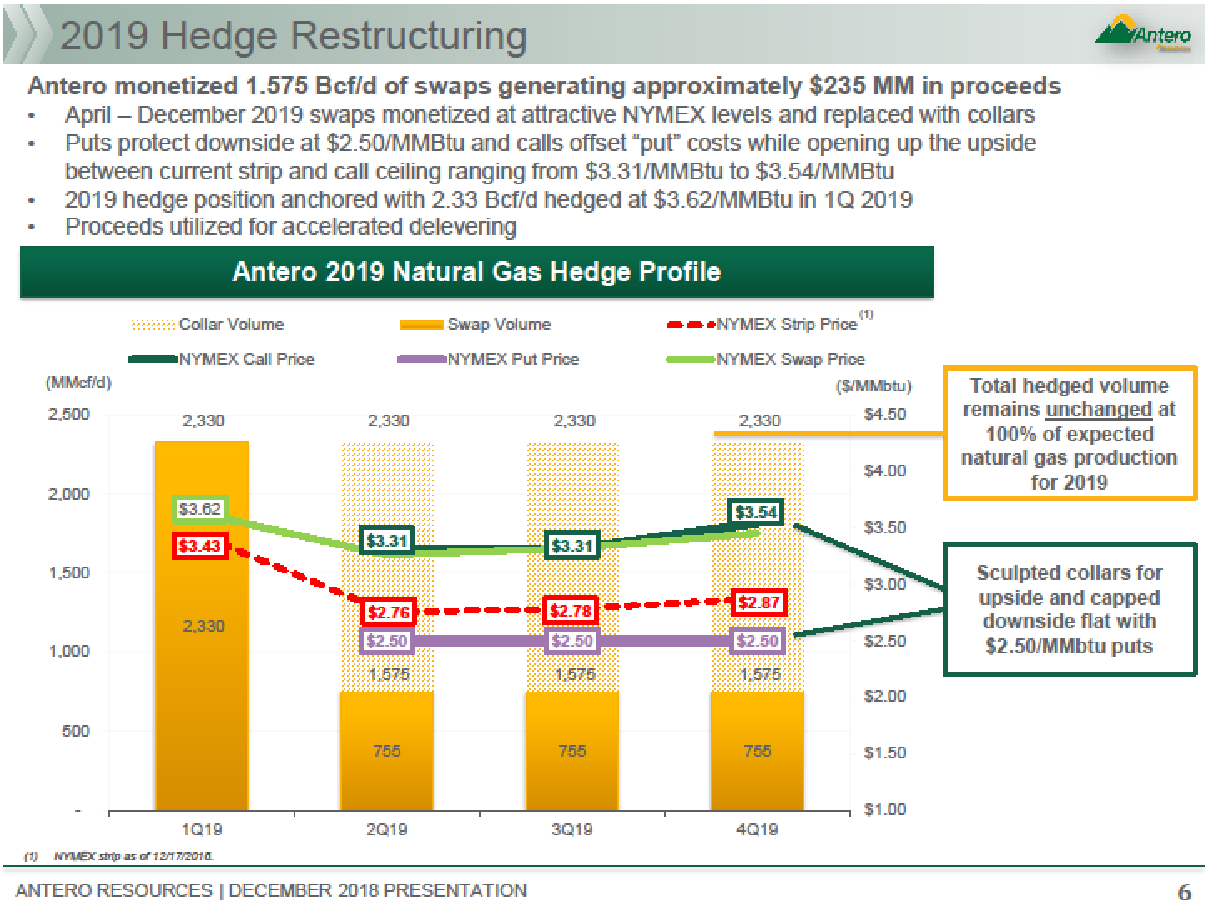
ANTERO RESOURCES: FINITE COLLAR OPTIONS[1] **9 FEB 2019**



INTRODUCTION

Mary Kirkpatrick, newly appointed Risk Manager at Antero Resources (AR), participated in the 2019 hedge restructuring in December 2018 that resulted in the collar hedge, illustrated above. Now Mary is concerned with the problem of what is the effect on the natural gas price (threshold) that justifies immediate investment (drilling) given the new hedging structure (if the hedge applies to proven undeveloped reserves)?

AR holds the E&D rights on approximately 620,000 net acres in West Virginia and Ohio, targeting the Marcellus WV and Utica Ohio gas shales. The December 2018 presentation

[1] © Dean A. Paxson, 2019. Parts of this case are from the AR 2017 10K and December 2018 Presentation, but the character is fictitious. The theoretical material is from Adkins et al. (2019), thanks to Roger Adkins, Paulo Pereira and Artur Rodrigues. Many of the numbers are the author's calculations. This case is not intended as an illustration of either good or bad business practices, and mixes hypothetical and actual data and names.

reported that the 3P (proven (PR), probable, possible) reserves are around 54.6 trillion cubic feet equivalent (Tcfe), with a PV10 $18.4B (PR $10.8B). There are some net 2370 undrilled core wells, which could be drilled over the next 16 years (at the current rate) with very high returns on the drilling at current oil and gas prices. Note ("SEC") proven reserves disclosed in the 10K 2017 were 17.3 Tcfe, (8.5 proven undeveloped, PUD).

Perhaps Mary will find the solution to her problem in recent articles on perpetual and finite collars, see Adkins and Paxson (2018) and Adkins et al. (2019).

Real Collar Option for an Investment Opportunity

1 Plain investment opportunity

AR has options to invest in several drilling projects whose value depends on a single source of uncertainty, that is natural gas at a price P, which is assumed to follow a geometric Brownian motion process:

$$dP = \alpha P dt + \sigma P dz, \tag{1}$$

where α and σ denote the risk-neutral drift and the volatility, respectively, and dz is an increment of the standard Wiener process. Additionally, $\alpha = r - \delta$, where r stands for the risk-free rate and δ is the convenience yield indicated in the term structure of futures prices. Assume the project requires an investment cost K, allowing AR to produce an output quantity Q. For the sake of simplicity, operating costs and taxes are not considered. After investing, the value of the active project is:

$$V(P) = \frac{PQ}{\delta}. \tag{2}$$

The value of an opportunity to invest in this project, $F(P)$, is given by:

$$F(P) = \begin{cases} (V(P^*) - K)\left(\frac{P}{P^*}\right)^{\beta_1} & \text{for } P < P^* \\ V(P) - K & \text{for } P \geq P^* \end{cases} \tag{3}$$

where P^* corresponds to the investment trigger:

$$P^* = \frac{\beta_1}{\beta_1 - 1}\frac{\delta}{Q}K, \tag{4}$$

and β_1 is the positive root of the characteristic quadratic equation $\frac{1}{2}\sigma^2\beta(\beta-1)+\alpha\beta-r=0$, i.e.,

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2}+\frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}. \tag{5}$$

2 The investment opportunity with a collar

Suppose that AR enters into a perpetual collar arrangement with Townbank that offers downside price protection in return for giving up some of the upside benefits. AR receives a minimum floor P_L, otherwise P, subject to a price cap P_H , where $P_H \geq P_L$. AR receives the instantaneous revenue $R(P, P_L, P_H, Q) = \min\{\max\{P_L, P\}, P_H\}Q$ where Q is the output volume. The collar corresponds to a contingent portfolio of options: a long call for $P < P_L$, a long put and a short call for $P_L \leq P < P_H$ and a short put for $P \geq P_H$.

2.1 Investments with perpetual collars

The solutions for an investment opportunity with such a perpetual collar are in Adkins and Paxson (2019). Ignoring any operating costs, $V_p(P)$ denotes the value of an active project whose output price P is bounded by a price floor P_L and a price cap P_H. The solution for $V_p(P)$ satisfies the following differential equation:

$$\frac{1}{2}\sigma^2P^2\frac{\partial^2 V_p(P)}{\partial P^2} + \alpha P\frac{\partial V_p(P)}{\partial P} - rV_p(P) + R(P) = 0, \tag{6}$$

where $R(P) = R(P, P_L, P_H, Q) = \min\{\max\{P_L, P\}, P_H\}\,Q$ for convenience, and $R(P) = P_LQ$ for $P < P_L$, $R(P) = PQ$ for $P_L \leq P < P_H$, $R(P) = P_HQ$ for $P \geq P_H$.

The general solution is: $V_P(P) = A_aP^{\beta_1} + A_bP^{\beta_2}$

where $$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(-\frac{1}{2}+\frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0. \tag{7}$$

The particular solutions depend where P stands in relation to P_L and P_H. Accordingly, the particular solution for $P < P_L$ is P_LQ/r, for $P \in [P_L, P_H)$ is PQ/δ, and for $P > P_H$ becomes

$P_H Q/r$. Considering that $V_p(0) = 0$, then $A_b = 0$ for $P < P_L$. Additionally, given that $V_p(P)$ has an upside limit of $P_H Q/r$ whenever $P \geq P_H$, then A_a must be set equal to 0 in this region. The solutions for all the regions are:

$$V_p(P) = \begin{cases} A_{11}P^{\beta_1} + \frac{P_L Q}{r} & \text{for } P < P_L \\ A_{21}P^{\beta_1} + A_{22}P^{\beta_2} + \frac{PQ}{\delta} & \text{for } P_L \leq P < P_H \\ A_{32}P^{\beta_2} + \frac{P_H Q}{r} & \text{for } P \geq P_H \end{cases} \tag{8}$$

The constants $A_{11}, A_{21}, A_{22}, A_{32}$ are found by ensuring that $V_p(P)$ is continuous and continuously differentiable along P. The solutions for the constants are:

$$\begin{aligned} A_{11} &= \frac{\left(P_H^{1-\beta_1} - P_L^{1-\beta_1}\right)Q}{\beta_1 - \beta_2}\left(\frac{\beta_2 - 1}{\delta} - \frac{\beta_2}{r}\right) \\ A_{21} &= \frac{P_H^{1-\beta_1}Q}{\beta_1 - \beta_2}\left(\frac{\beta_2 - 1}{\delta} - \frac{\beta_2}{r}\right) \\ A_{22} &= -\frac{P_L^{1-\beta_2}Q}{\beta_1 - \beta_2}\left(\frac{\beta_1 - 1}{\delta} - \frac{\beta_1}{r}\right) \\ A_{32} &= \frac{\left(P_H^{1-\beta_2} - P_L^{1-\beta_2}\right)Q}{\beta_1 - \beta_2}\left(\frac{\beta_1 - 1}{\delta} - \frac{\beta_1}{r}\right) \end{aligned} \tag{9,10,11,12}$$

The value of the option to invest in a project with a perpetual collar, $F_p(P)$ must satisfy the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 F_p(P)}{\partial P^2} + \alpha P \frac{\partial F_p(P)}{\partial P} - rF_p(P) = 0. \tag{13}$$

The general solution has the form $F_p(P) = B_a P^{\beta_1} + B_b P^{\beta_2}$. Considering that $F_p(0) = 0$ then $B_b = 0$. The arbitrary constant B_a is found using the value matching condition (VM):

$$F_p(P_p^*) = B_a {P_p^*}^{\beta_1} = V_p(P_p^*) - K, \tag{14}$$

i.e.,

$$B_a = \left(V_p(P_p^*) - K\right)\left(\frac{1}{P_p^*}\right)^{\beta_1},$$

and the investment trigger, P_p^*, is obtained by solving the smooth pasting condition (SP):

$$\beta_1 B_a {P_p^*}^{\beta_1 - 1} = V'_p(P_p^*).$$

Multiplying both sides by P_p^* and using the VM condition:

$$\beta_1(V_p(P_p^*) - K) = V'_p(P_p^*)P_p^* \qquad (15)$$

The price floor must be lower than Kr/Q, otherwise it produces a risk-free profit (a positive value for every P). The trigger is:

$$P_p^* = \left(\frac{\beta_1}{(\beta_1-\beta_2)A_{32}}\left(K - \frac{P_H Q}{r}\right)\right)^{\frac{1}{\beta_2}} > P_H, \qquad \text{for } K \geq K_p^H, \qquad (16)$$

where

$$K_p^H = \frac{P_H^{\beta_2}}{\beta_1}\left(P_H^{1-\beta_2} - P_L^{1-\beta_2}\right)Q\left(\frac{\beta_1 - 1}{\delta} - \frac{\beta_1}{r}\right) + \frac{P_H Q}{r}.$$

The solution for $F_p(P)$ is:

$$F_p(P) = \begin{cases} \left(V_p(P_p^*) - K\right)\left(\frac{P}{P_p^*}\right)^{\beta_1} & \text{for } P < P_p^* \\ V_p(P) - K & \text{for } P \geq P_p^* \end{cases} \qquad (17)$$

2.2 Investments with finite-lived collars

In December 2018, AR established a type of finite-lived collar by buying put options on natural gas futures with an exercise price at around \$2.50/MMbtu and selling call options with an exercise price of around \$3.31 (2Q2019), which amounts to a collar that guarantees a minimum price P_L but limits its gains to a maximum P_H, but which is confined to a finite duration $T < \infty$ years.

The value of a project protected by a collar that lasts for T years corresponds to a package of European collars. It is equivalent to a portfolio that includes: (i) a long position in a perpetual collar, (ii) a short position in a forward-start perpetual collar (that starts after T years). Additionally, if the project lasts beyond the collar period, AR will have (iii) a long position in the expected profits that will start after T years. Combining (i) and (ii) replicates the finite-collar, whereas (iii) captures the value in operating the project without restrictions in P perpetually (or for the remaining project life) after the end of the collar.

Accordingly, the value of an active project with a finite-lived collar is given by:

$$V_f(P) = V_p(P) - S(P) + \frac{PQ}{\delta}e^{-\delta T}. \qquad (18)$$

The first term, $V_p(P)$ is (8). The second term, $S(P)$, represents the forward-start perpetual collar (a collar that starts in the future moment T), which is given by:

$$\begin{aligned} S(P) \quad &= A_{11}P^{\beta_1}N(-d_{\beta_1}(P,P_L)) + \frac{P_LQ}{r}e^{-rT}N(-d_0(P,P_L)) \\ &+A_{21}P^{\beta_1}\big(N(d_{\beta_1}(P,P_L)) - N(d_{\beta_1}(P,P_H))\big) \\ &+A_{22}P^{\beta_2}\big(N(d_{\beta_2}(P,P_L)) - N(d_{\beta_2}(P,P_H))\big) \\ &+\frac{PQ}{\delta}e^{-\delta T}(N(d_1(P,P_L)) - N(d_1(P,P_H))) \\ &+A_{32}P^{\beta_2}N(d_{\beta_2}(P,P_H)) + \frac{P_HQ}{r}e^{-rT}N(d_0(P,P_H)), \end{aligned} \tag{19}$$

where $N(.)$ is the standard normal cumulative distribution, and

$$d_\beta(P,x) \quad = \frac{\ln P - \ln x + \left(r-\delta+(\beta-0.5)\sigma^2\right)T}{\sigma\sqrt{T}}, \quad \beta \in \{0,1,\beta_1,\beta_2\},\ x \in \{P_L,P_H\}. \tag{20}$$

In (18) the negative sign represents a short position in the forward-start perpetual collar, while the last term is the present value of the expected profits that will start after T.

The value of the option to invest in a project with a finite collar, $F_f(P)$ must satisfy the following ordinary differential equation:

$$\frac{1}{2}\sigma^2P^2\frac{\partial^2F_f(P)}{\partial P^2} + \alpha P\frac{\partial F_f(P)}{\partial P} - rF_f(P) = 0, \tag{21}$$

whose general solution is $F_f(P) = C_aP^{\beta_1} + C_bP^{\beta_2}$. Considering the boundary at $P = 0$ ($F_f(0) = 0$), so $C_b = 0$. The arbitrary constant C_a and the investment trigger P_f^* are found using the value VM and SP conditions:

$$\begin{aligned} C_a{P_f^*}^{\beta_1} &= V_f(P_f^*) - K \\ \beta_1C_a{P_f^*}^{\beta_1-1} &= V'_f(P_f^*). \end{aligned} \tag{22}$$

where $V_f(P_f^*)$ is (18) with $P = P_f^*, and\ V'_f(P_f^*)$ is the first derivative of (18).

For the finite collar, no restrictions are required for the VM and SP, meaning that the transition between the idle and the active stages can occur for any P $\left(P_L \lesseqgtr P_f^* \lesseqgtr P_H\right)$. However, the price floor must be lower than $Kr/Q(1-e^{-rT})$, otherwise it produces a risk-free profit. The SP condition is used to find the trigger P_f^* and can be reduced to:

$$\beta_1(V_f(P_f^*) - K) - V'_f(P_f^*)P_f^* = 0. \tag{23}$$

The value of the option to invest in the project granted with a finite-lived collar (F_f) is:

$$F_f(P) = \begin{cases} (V_f(P_f^*) - K)\left(\frac{P}{P_f^*}\right)^{\beta_1} & \text{for } P < P_f^* \\ V_f(P) - K & \text{for } P \geq P_f^* \end{cases} \quad (24)$$

3.1 Easy Spreadsheets for Investments with perpetual and finite collars

These investment thresholds and real option values are easily calculated using spreadsheets (with some items for finite collars appearing much lower in the Excel sheet, for convenience). The solution for the investment threshold and value without a collar is completely analytical as shown in B13 and B11.

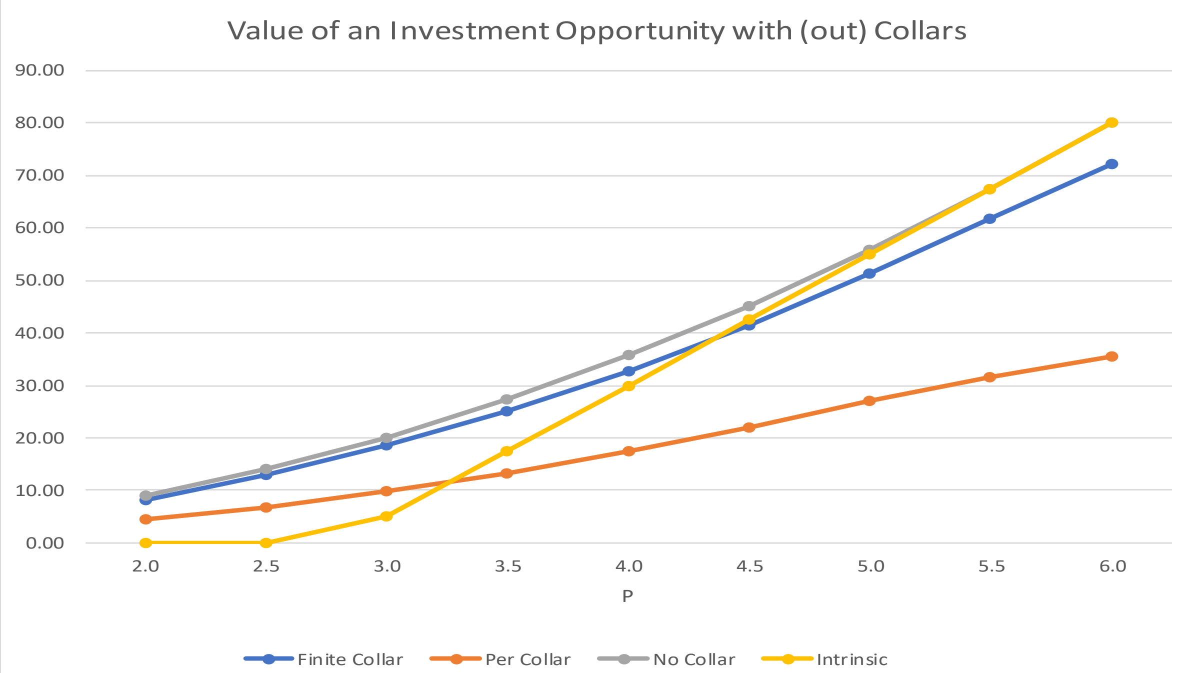
	A	B	C	D
1	**INVESTMENT OPPORTUNITY FOR A PEP WITH A PERPETUAL COLLAR OPTION**			
2	**INPUT**			CASE EQS
3	P	2.00		
4	K	70.00		
5	σ	0.20		
6	r	0.04		
7	δ	0.04		
8	PL	2.00		
9	PH	6.00		
10	**OUTPUT**			
11	**ROV PLAIN CALL**	8.9286	IF(B3<B13,((B13/B7-B4)*(B3/B13)^B14),B12)	3
12	P/δ-K	0.0000	MAX(B3/B7-B4,0)	2
13	**P^ NO COLLAR**	**5.6000**	(B14/(B14-1))*B4*B7	4
14	β_1	2.0000	0.5-(B6-B7)/(B5^2)+SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	5
15	β2	-1.0000	0.5-(B6-B7)/(B5^2)-SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	7
16				
17	**PEP COLLAR**			
18	AC21	-1.3889	(B9/(B9^B14))*(B20/B22)	10
19	AC22	33.3333	(-B8/(B8^B15))*(B21/B22)	11
20	[ ]	-0.0400	(B6*B15-B6-B7*B15)	
21	( )	-0.0400	(B6*B14-B6-B7*B14)	
22	{ }	0.0048	(B14-B15)*B6*B7	
23	AC21 P^${}^{\beta 1}$	-5.5556	B18*(B3^B14)	10
24	AC22 P^${}^{\beta 2}$	16.6667	B19*(B3^B15)	11
25	VpC(P*)	94.5297	B29/B7+B18*(B29^B14)+B19*(B29^B15)	8
26	V'pC(P*)	10.3075	1/B7+B14*B18*(B29^(B14-1))+B15*B19*(B29^(B15-1))	
27	VpC(P>P*)	61.1111	B3/B7+B18*(B3^B14)+B19*(B3^B15)	8
28	P*	0.0000	B14*(B25-B4)-B26*B29	15
29	**P* PEP COLLAR**	**4.7596**	SOLVER: Set B28=0, Changing B29	
30	**ROV PEP COLLAR**	4.3312	IF(B3<B29,((B25-B4)*(B3/B29)^B14),B27-B4)	17
31	ODE ROV PEP C	0.0000	0.5*(B5^2)*(B3^2)*B33+(B6-B7)*B3*B32-B6*B30	13
32	VC Δ	4.3312	(B14*(B25-B4)*((B3^(B14-1))/(B29^B14)))	
33	VC Γ	2.1656	(B14*(B14-1)*(B25-B4)*((B3^(B14-2))/(B29^B14)))	
34	ROV	8.9286	IF(B3<B13,((B13/B7-B4)*(B3/B13)^B14),B12)	
35	ODE ROV NO C	0.0000	0.5*(B5^2)*(B3^2)*B37+(B6-B7)*B3*B36-B6*B34	
36	VC Δ	8.9286	(B14*(B13/B7-B4)*((B3^(B14-1))/(B13^B14)))	
37	VC Γ	4.4643	(B14*(B14-1)*(B13/B7-B4)*((B3^(B14-2))/(B13^B14)))	

The solution for the investment threshold and value with a perpetual collar is quasi-analytical, obtained by using SOLVER, setting B28=0, changing B29. The solution for the investment threshold and value with a finite collar is more complex, and is obtained by setting B31=0, changing B32 using SOLVER. Note all of these examples assume P_L<P, P*, P**<P_H. Note that all of these solutions can be extended to show the sensitivity of thresholds and values to changes in the parameter values by copying the columns across (remembering to also copy across the "hidden" items in the finite collar spreadsheet).

	A	B	C	D
1	EASY SPREADSHEET FOR A PERPETUAL PROJECT WITH A FINITE COLLAR CASE Jan 2019			CASE Eqs
2	T	10		
3	P	2.00		
4	K	70.00		
5	σ	0.20		
6	r	0.04		
7	δ	0.04		
8	P_L	**2**		
9	P_H	**6**		
10	**OUTPUT**			
11	VC	61.1111		8
12	VC PV	50.0000	IF(B3<B8,B8/B6,IF(B3>B9,B9/B6,B3/B7))	
13	P/δ	50.0000	B3/B7	
14	β_1	2.0000	0.5-(B6-B7)/(B5^2)+SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	5
15	β2	-1.0000	0.5-(B6-B7)/(B5^2)-SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	7
16	A11*P^b1	11.1111		9
17	A21*P^b1	-5.5556		10
18	A22*P^b2	16.6667		11
19	A32*P^b2	-133.3333		12
20	VC(P**)	99.1442		8
21	VfC(P)	52.5734	B11-B23+(B3/B7)*EXP(-(B7)*B2)	18
22	PV at Forward End	33.5160	(B3/B7)*EXP(-(B7)*B2)	
23	S(P)	42.0537		19
24	VfC(P**)	126.1367		
25	PV at Forward End	87.6640		
26	S(P**)	60.6715		
27	V'fC(P**)	21.4624	B28+B29-B30	
28	V'C(P**)	9.2509		
29	PV at Forward End	16.7580	(1/B7)*EXP(-(B7)*B2)	
30	S'(P**)	4.5465		
31	P**	0.0000	B14*(B24-B4)-B27*B32	23
32	P**	**5.2312**	Solver: Set B31=0, changing B32.	23
33	ROV FC	**8.2056**	**IF(B3<B32,((B24-B4)*(B3/B32)^B14),B21-B4)**	24
34	ODE ROV FC	0.0000	0.5*(B5^2)*(B3^2)*B36+(B6-B7)*B3*B35-B6*B33	21
35	VC Δ	8.2056	(B14*(B24-B4)*((B3^(B14-1))/(B32^B14)))	
36	VC Γ	4.1028	(B14*(B14-1)*(B24-B4)*((B3^(B14-2))/(B32^B14)))	

A typical concern is how the real option value changes with changes in the underlying price (assuming everything else including the quantity of production, 1 in this easy example, and K remain constant).

Finite Collar	**8.21**	**12.82**	**18.46**	**25.13**	**32.82**	**41.54**	**51.28**	**61.85**	**72.15**
Per Collar	**4.33**	**6.77**	**9.75**	**13.26**	**17.32**	**21.93**	**26.94**	**31.55**	**35.56**
No Collar	**8.93**	**13.95**	**20.09**	**27.34**	**35.71**	**45.20**	**55.80**	**67.52**	**80.00**
Intrinsic	**0.00**	**0.00**	**5.00**	**17.50**	**30.00**	**42.50**	**55.00**	**67.50**	**80.00**
P	2	2.5	3	3.5	4	4.5	5	5.5	6



While the natural gas price that justifies immediate investment does not change with P, the value of the investment opportunity changes significantly. At high prices, AR would benefit by not having established a price collar through floor and ceiling options. If expecting high future prices, the next best arrangement would be a finite collar with a short duration (T=10 in this example), and the worst arrangement would be a perpetual collar, giving up all of the upside forever.

PROJECT QUESTIONS

1. **After the AR Valentine's Day disclosures, help Mary understand the exposure of AR to NG (and NGL) price changes, comparing no collar, finite and perpetual collars. Note the price limits in the case are hypothetical along with Q and K.**
2. **What are the recalculated volatilities and current NG (and NGL) price levels, based on your reasonable assumptions, updated to Feb 2019?**
3. **Is AR right to establish a collar for the rest of 2019?**
4. **Illustrate the sensitivities of no, finite and perpetual collars to changes in the parameter values you consider the most important. Can these possible changes be "hedged"?**

REFERENCES

Adkins, R. and D. Paxson (2018) “Real Collars as Alternative Incentives for Subsidizing Energy Facilities”, The Manchester School, forthcoming October.

Adkins, R., D. Paxson, P. Pereira, and A. Rodrigues (2018), “Investment Decisions with Finite-lived Collars”, presented at the Real Options Conference, Düsseldorf, June, revision submitted to the Journal of Economic Dynamics and Control, January 2019.